

Pulse propagation, population transfer and light storage in five-level media

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We consider adiabatic interaction of five-level atomic systems and their media with four short laser pulses under the condition of all two-photon detunings being zero. We derive analytical expressions for eigenvalues of the system's Hamiltonian and determine conditions of adiabaticity for both the atom and the medium. We analyse, in detail, the system's behaviour when the eigenvalue with non-vanishing energy is realized. As distinct from the usual dark state of a five-level system (corresponding to zero eigenvalue), which is a superposition of three states, in our case the superposition of four states does work. We demonstrate that this seemingly unfavourable case nevertheless completely imitates a three-level system not only for a single atom but also in the medium, since the propagation equations are also split into those for three- and two-level media separately. We show that, under certain conditions, all the coherent effects observed in three-level media, such as population transfer, light slowing, light storage, and so on, may efficiently be realized in five-level media. This has an important advantage that the light storage can be performed twice in the same medium, i.e., the second pulse can be stored without retrieving the first one, and then the two pulses can be retrieved in any desired sequence.

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I. INTRODUCTION

Coherent interaction of light signals with quantum systems attracted considerable interest for their importance in both fundamental science and practical applications. A prominent example of coherent interactions is electromagnetically induced transparency (EIT) [1–3] which can be used to eliminate the resonant absorption of a laser beam incident upon a coherently driven medium with appropriate energy levels. EIT technique allows controlled manipulations of the optical properties of atomic or atom-like media via coupling them with signal and control fields. In particular, it is possible to greatly slow down the optical (signal) pulse [4–6] and even stop it to attain reversible storage and retrieval of information [7–9].

Despite a huge number of publications, light storage remains in the focus of attention of researchers, since it is one of key components in optical (quantum) information processing [1, 10–14]. Another application of coherent interactions is controllable population transfer between the atomic levels and constructing desired coherent superpositions of different states [15–17]. These effects are also employed widely in such fields of research as laser cooling of atoms, lasing without inversion, new precision techniques of magnetometry, coherent control of chemical reactions, and so on.

All the above-listed phenomena are comprehensively studied, both theoretically and experimentally, for var-

ious three level systems and their media [18–24]. Although multilevel atomic and atom-like systems do not provide new physical principles in addition to quantum interference and principle of superposition, they widen essentially the possibilities of experimental realizations and practical applications. The idea of a double-EIT (DEIT) regime is introduced in [25] and modified in [26]. The laser cooling scheme for trapped atoms or ions which is based on DEIT is discussed in [27]. DEIT in a medium, consisting of four level atoms in the inverted-Y configuration is discussed in [28]. DEIT in a ring cavity is studied in [29]. Enhanced cross-phase modulation based on DEIT is reported in [30]. Work [31] examines dark state polariton formation in a four-level system. Quantum memory for light via stimulated off-resonant Raman process is considered in [32] beyond the three-level approximation. Work [33] proposes, through numerical calculations, to use multilevel systems involving hyperfine structure in problems of localization of excitations via dark state formation in the EIT processes. Work [34] investigates five-level atoms and media driven by four light pulses in nonadiabatic regime. Two of four pulses are assumed weak and treated as perturbation in the first order. Work [35] observed experimentally off-resonance EIT-based group delay in multilevel D_2 transition in rubidium. Enhancement of EIT in a double-lambda system in cesium atomic vapor by specific choice of atomic velocity distribution is observed in [36]. A scheme based on two sequential STIRAP processes with four laser fields is proposed in [37] for measurement of a qubit of two magnetic sublevels of the ground state of alkaline-earth metal ions. Another topic where multilevel systems were used was generalization of the notion of dark-state polariton

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[38] and discussion of possibility to apply multilevel EIT to quantum information processing.

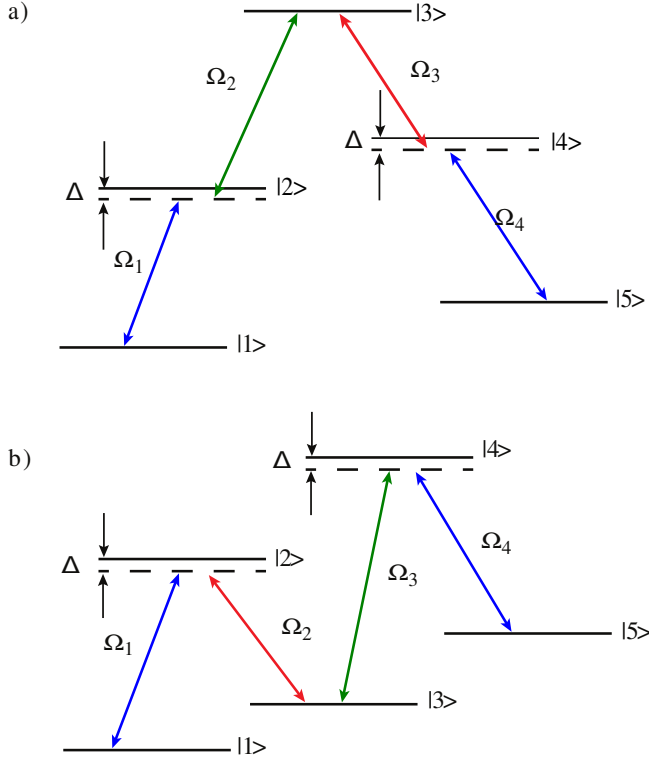


FIG. 1. Five-level coupling schemes: (a) Extended Λ -scheme, (b) M-type scheme.

In the present paper we study both analytically and numerically a five-level atomic system interacting adiabatically with four co-propagating laser pulses of different durations and different sequences of turning on and off. We require that each laser pulse interacts (be resonant) with only one of the adjacent transitions, and assume all the two-photon detunings to be zero. Two examples of such level diagrams are shown in Fig. 1. Another example is the ladder-system which may turn out to be rather useful for problems of excitation of Rydberg states.

As distinct from all above-sited works, we concentrate on those eigenstates of interaction Hamiltonian (see below) whose eigenvalues are different from zero. We will show that these eigenstates are similar to the well-known dark and bright states in a three-level system. As distinct from the state in the M-system considered in [15], the levels 2 and 4 are at interaction with laser fields populated, but the population of level 3 remains zero. We will demonstrate for our case that an efficient and more flexibly controllable population transfer and light storage becomes possible. We also study the advantages of this technique. Specifically, we show the possibility of successive storage of two pulses with their subsequent retrieval. The first pulse is stored into the coherence ρ_{51} , which exactly reproduces, after turning off the interaction, the shape of the pulse Ω_2 . Since the coherence ρ_{31} remains zero during all time of interaction, the same medium can

be used again for storage of another pulse.

The paper is organized as follows: In section II we derive eigenfunctions and eigenvalues of systems under consideration and discuss the relevant cases. In section III we study adiabatic population transfer in five-level systems. Section IV derives the equations of propagation and presents their analytical solution. In the same section the regime of adiabaton is demonstrated. In section V we show the possibility to store optical information in considered media. We conclude with a final discussion in section VI.

II. EIGENFUNCTIONS AND EIGENVALUES OF INTERACTION HAMILTONIAN

Consider a five-level atomic system as shown in Fig. 1. Four (in general) laser pulses are close to resonance with respective transitions (Fig. 1). Hamiltonian of interaction in rotating-wave approximation, and under the assumptions that the carrier frequencies of laser pulses are tuned near resonance with one of the adjacent atomic transitions, and pulse durations are much shorter compared to relaxation times in the system, has the following form:

$$H = \sum_i \sigma_{i,i} \delta_{i-1} - \left(\sum_i \sigma_{i,i+1} \Omega_i + h.c. \right), \quad (1)$$

with the projection matrices $\sigma_{ij} = |i\rangle\langle j|$, the Rabi frequencies Ω_i at transitions $i \rightarrow i+1$, and δ_{i-1} representing $(i-1)$ -photon detunings (with $\delta_0 = 0$). The Rabi frequencies are assumed to be real and positive. Phases, which can vary during propagation, are included in the single-photon detunings ($\Delta_i = \omega_{i+1,i} - \omega_i + \varphi_i$, if $\omega_{i+1,i} > 0$ and $\Delta_i = \omega_{i,i+1} - \omega_i + \varphi_i$ if $\omega_{i+1,i} < 0$). Definition of multi-photon detunings depends on the specific scheme of interaction. For an M-system (see Fig. 1(b)) the multi-photon detunings are $\delta_2 = \Delta_1 - \Delta_2$, $\delta_3 = \Delta_3 + \delta_2$, $\delta_4 = \Delta_4 - \delta_3$. For an extended Λ -system (see Fig. 1(a)), the multi-photon detunings are $\delta_2 = \Delta_1 + \Delta_2$, $\delta_3 = -\Delta_3 + \delta_2$, $\delta_4 = -\Delta_4 + \delta_3$.

Eigenvalues of the Hamiltonian (1) can easily be derived analytically if all of the two-photon detunings are zero, i.e.

$$\delta_2 = 0, \delta_3 - \delta_1 = 0, \delta_4 - \delta_2 = 0. \quad (2)$$

For an M-system, these conditions mean equal single-photon detunings, while for the extended Λ -system the single-photon detunings have equal absolute values, but differ in sign (see Fig. 1). When conditions (2) are met, one of five eigenvalues of the Hamiltonian is $\lambda = 0$. The detailed calculations of the remaining four eigenvalues are presented in Appendix A.

Consider now a special case, when the pulses Ω_1 and Ω_4 coincide by their temporal profiles (but the frequencies and phases of pulses may be different). In this case, the

eigenvalues of the Hamiltonian (1) are

$$\begin{aligned}\lambda_0 &= 0, \\ \lambda_{1,3} &= \frac{1}{2} \left(\Delta \mp \sqrt{\Delta^2 + 4\Omega_1^2} \right), \\ \lambda_{2,4} &= \frac{1}{2} \left(\Delta \mp \sqrt{\Delta^2 + 4(\Omega_1^2 + \Omega_2^2 + \Omega_3^2)} \right)\end{aligned}\quad (3)$$

We note, that when the fields are turned off, we get $\lambda_{1,2} \rightarrow 0$ and $\lambda_{3,4} \rightarrow \Delta$. It should be emphasized that the eigenvalues $\lambda_{1,3}$ depend upon only the field Ω_1 and coincide with the eigenvalues of a two-level system, driven by field Ω_1 . Similarly, the eigenvalues $\lambda_{2,4}$ are equal to the eigenvalues of a two-level system, driven by an effective field $(\Omega_1^2 + \Omega_2^2 + \Omega_3^2)^{1/2}$. Adiabatic evolution requires the following conditions to be met (see Appendix A for details):

$$\begin{aligned}\Delta T &\gg 1, \\ \frac{(\Omega_2^2 + \Omega_3^2)T}{\Delta} &\gg 1, \\ \frac{\Omega_1^2 T}{\Delta} &\gg 1\end{aligned}\quad (4)$$

with the duration T of the shortest pulse. The first condition mirrors the adiabaticity condition for a two-level system. The second condition corresponds to the adiabaticity condition for a three-level system. The third condition is only relevant in the time interval where all pulses overlap (i.e., when $\Omega_2^2 + \Omega_3^2 \neq 0$).

To write the eigenvectors corresponding to the eigenvalues λ_1 and λ_2 we introduce the following notations:

$$\begin{aligned}\Omega^2 &= \Omega_2^2 + \Omega_3^2, \tan \theta = \frac{\Omega_2}{\Omega_3}, \\ \tan \Phi_1 &= -\frac{\lambda_1}{\Omega_1}, \tan \Phi_2 = -\frac{\lambda_2}{\Omega_1}, \\ \tan \Phi &= -\frac{\Omega}{\Omega_1} \cos \Phi_2\end{aligned}\quad (5)$$

Then, eigenvector corresponding to the eigenvalue λ_1 is

$$|\lambda_1\rangle = |\psi_1\rangle \cos \theta - |\psi_2\rangle \sin \theta \quad (6)$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are superposition states of two-level systems $1 \rightarrow 2$ and $5 \rightarrow 4$:

$$\begin{aligned}|\psi_1\rangle &= \cos \phi_1 |1\rangle - \sin \phi_1 |2\rangle \\ |\psi_2\rangle &= \cos \phi_1 |5\rangle - \sin \phi_1 |4\rangle\end{aligned}\quad (7)$$

It is apparent that the eigenvector corresponding to the λ_1 does not involve state $|3\rangle$ and is equal to the dark state of a three-level Λ -system, if we replace the lower states by the superposition states $|\psi_1\rangle$ and $|\psi_2\rangle$.

Similarly, the eigenvector corresponding to the eigenvalue λ_2 yields

$$|\lambda_2\rangle = |\psi'_1\rangle \cos \Phi \sin \theta - \sin \phi |3\rangle + |\psi'_2\rangle \cos \Phi \cos \theta, \quad (8)$$

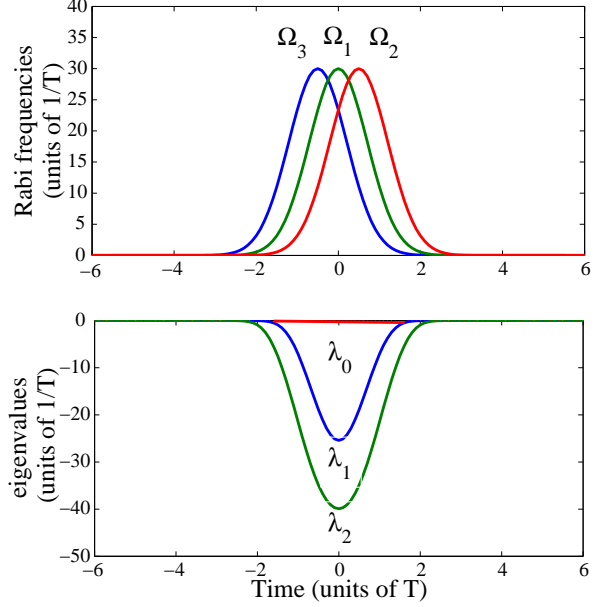


FIG. 2. Time dependencies of the pulses (top) and the adiabatic energies (bottom) for considered systems. Shapes of all pulses are Gaussian. The single-photon detuning is $\Delta = 10/T$. It is seen clearly that in the range of overlapping of the pulses, the adiabaticity of interaction is ensured.

where

$$\begin{aligned}|\psi'_1\rangle &= \cos \Phi_2 |1\rangle - \sin \Phi_2 |2\rangle \\ |\psi'_2\rangle &= \cos \Phi_2 |5\rangle - \sin \Phi_2 |4\rangle\end{aligned}\quad (9)$$

As in the previous case, the eigenvector $|\lambda_2\rangle$ is equal to that of the bright state of a three-level Λ -system [39, 40], if we replace the lower states by superposition states $|\psi'_1\rangle$ and $|\psi'_2\rangle$. The time behavior of eigenvalues λ_i in the special case above for different pulse sequences is demonstrated in Fig. 2 and Fig. 3

III. POPULATION TRANSFER

As follows from the expressions (6) and (8) the five-level system imitates the three-level lambda system. Thus, we can use the state $|\lambda_1\rangle$ to transfer the system from state $|1\rangle$ to state $|5\rangle$ by a STIRAP-like process, driven by the pulse sequence introduced above (see Fig. 4). In contrast to a simple three-level Λ -system, during the interaction some transient population shows up in the intermediate levels $|2\rangle$ and $|4\rangle$ of the five-level system. However, these transient populations are very small, if the one-photon detuning is sufficiently large, but still satisfies the adiabaticity condition (3). It should be noted that the condition of large one-photon detuning is not very crucial for the population transfer. The dynamics of populations in the described case is shown in Fig. 4.

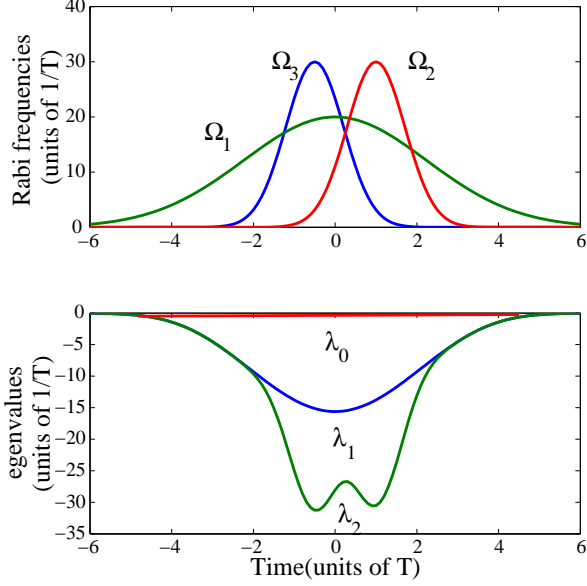


FIG. 3. Same as in Fig.2, but for another sequence of pulses.

Similarly, the state $|\lambda_2\rangle$ is analogous to the bright state of a lambda-system and we can use these states for adiabatic transfer from state $|5\rangle$ to state $|1\rangle$ by a b-STIRAP-like process [39, 40] driven by the pulse sequence in Fig. 3, because the state $|\lambda_2\rangle$ is not realized with the pulse sequence of Fig. 2, according to the definition of the angle Φ . We emphasize that the STIRAP-technique is applicable for both schemes of pulse sequence in Figs. 2 and 3. The dynamics of populations in the b-STIRAP case are demonstrated in Fig. 5. Note that the two eigenstates $|\lambda_1\rangle$ and $|\lambda_2\rangle$ render the five-level system, driven by a considered pulse sequence, fully reversible. Thus, we can transfer atomic population from state $|1\rangle$ to state $|5\rangle$ by a STIRAP-like process and from state $|5\rangle$ to state $|1\rangle$ by a b-STIRAP-like process with the same sequence of pulses.

IV. MEDIUM OF ATOMS

Now we move from a single-atom case to that of a medium consisting of the described atoms. We start from the well-known truncated Maxwell equation in running coordinates x , $\tau = t - x/c$:

$$\frac{\partial E_i}{\partial x} = i \frac{2\pi\omega_i}{c} N d_i \quad (10)$$

Here E_i are the complex amplitudes of electric fields of the pulses, N is the number density of medium atoms, and d_i are the amplitudes of induced dipole moments of each individual atom at a frequency ω_i , $\langle\psi|d|\psi\rangle = \sum d_i \exp(-i\omega_i\tau) + c.c..$ These amplitudes can be ex-

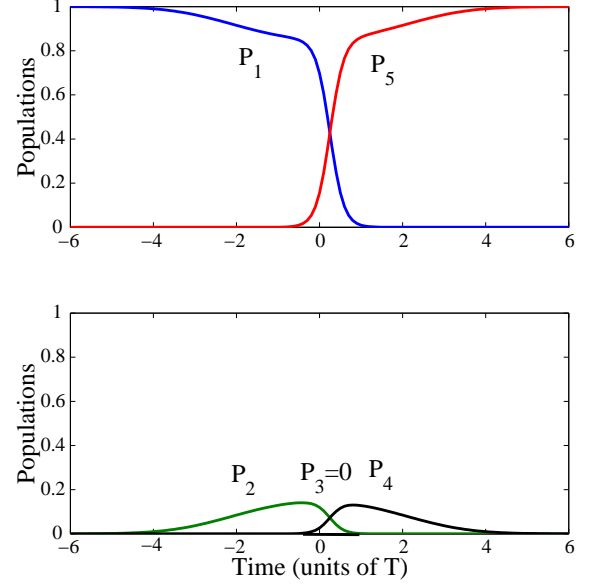


FIG. 4. Dynamics of the population transfer from initial state $|1\rangle$ to final state $|5\rangle$ if the atom is in state $|\lambda_1\rangle$ and the pulse sequence is the same as in Fig. 3. ($P_i = |\langle i|\lambda_1\rangle|^2$). The single photon detuning is $\Delta = 20/T$.

pressed in terms of the amplitudes of atomic populations b_i of bare states and the matrix elements of the dipole moment $\langle i|d|i+1\rangle$: $d_i = b_i^* b_{i+1} \langle i|d|i+1\rangle$ if $\omega_{i+1,i} > 0$ and $d_i = b_i b_{i+1}^* \langle i|d|i+1\rangle$ if $\omega_{i+1,i} < 0$. The coefficients b_i are determined by the non-stationary Schroedinger equation with Hamiltonian (1).

Separating real and imaginary parts in the truncated equation of propagation, differentiating the equation for the phase with respect to time, and combining the obtained equations with the Schroedinger equation, we obtain in the general case a self-consistent system of equations describing variation of frequencies (one-photon detunings) and intensities (Rabi frequencies) of pulses during propagation in medium. For example, in the case of medium consisting of M-type atoms we obtain:

$$\begin{aligned} \frac{\partial \Omega_1^2}{\partial x} &= q_1 \frac{\partial |b_1|^2}{\partial \tau} \\ \frac{\partial \Omega_2^2}{\partial x} &= -q_2 \frac{\partial (|b_1|^2 + |b_2|^2)}{\partial \tau} \\ \frac{\partial \Omega_3^2}{\partial x} &= -q_3 \frac{\partial (|b_4|^2 + |b_5|^2)}{\partial \tau} \\ \frac{\partial \Omega_4^2}{\partial x} &= q_5 \frac{\partial |b_5|^2}{\partial \tau} \\ \frac{\partial \Delta_i}{\partial x} &= q_i \frac{\partial}{\partial \tau} \frac{\text{Re}(b_i^* b_{i+1})}{\Omega_i} \end{aligned} \quad (11)$$

where $q_i = 2\pi N \omega_i |d_{i,i+1}|^2 / \hbar c$ is the propagation constant. In the case of an extended Λ -system (Fig. 1a)

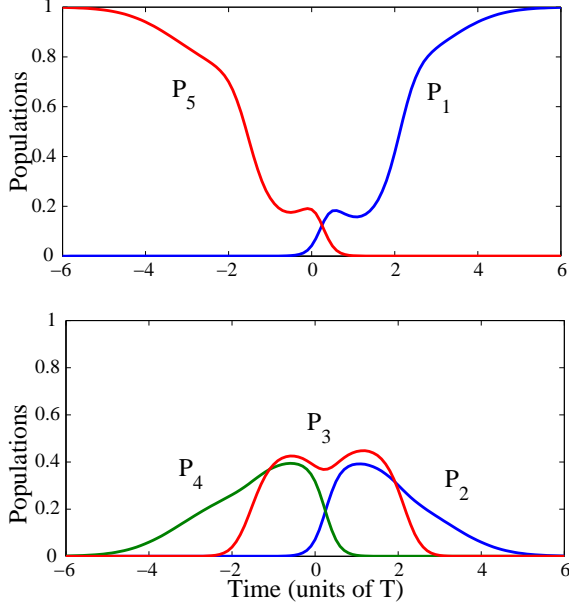


FIG. 5. Dynamics of the population transfer from initial state $|5\rangle$ to final state $|1\rangle$ if the atom is in state $|\lambda_2\rangle$. The pulse sequence and parameters are the same as in Fig. 4 ($P_i = |\langle i|\lambda_2\rangle|^2$).

the equations remain essentially the same, but we must change the signs of the rhs in the second and third equations. As follows from equations (11), during propagation in the medium not only the shapes of pulses may vary essentially, but also the conditions for detuning of resonances may be violated. Variations of resonance detunings are caused by processes of self-phase modulation (parametric broadening of pulse spectrum [41]). The modification of the shapes of pulses is caused both by the nonlinear group velocity (which can result in formation of shock wavefronts [42]) and by energy transfer between the pulses (which can lead to full depletion of one of pulses [43]). It is, however, obvious that all these processes are proportional to the length of propagation. Hence, if the optical length of the medium is sufficiently short, the variations of detunings and intensities can be negligibly small.

Since it is only the time derivatives that enter the right-hand sides of the equations (11), we can use expressions (6) and (8) for the atomic amplitudes in these equations and this will be equivalent to allowance for the first non-adiabatic corrections. Correspondingly, the conditions of smallness of rhs of (11) serve as a criteria of insignificance of changes in spatial and temporal characteristics of pulses and thus a criteria of adiabaticity of the interaction in the medium. For simplicity we will restrict ourselves to the case of equal oscillator strengths in all transitions (in the case of different oscillator strengths, we can proceed as in the three-level system [44]). In case

of the state $|\lambda_1\rangle$ we now obtain from (11)

$$\begin{aligned} \frac{\partial(\Omega_1^2 + \Omega_4^2)}{\partial x} &= q \frac{\partial \cos^2 \Phi_1}{\partial \tau}, \\ \frac{\partial(\Delta_1 + \Delta_4)}{\partial x} &= -q \frac{\partial \cos(2\Phi_1)}{\partial \tau} \frac{1}{\Delta}, \\ \frac{\partial \Delta_2}{\partial x} &= -\frac{\partial \Delta_3}{\partial x} = 0, \\ \frac{\partial \Omega^2}{\partial x} &= 0, \\ \frac{\partial \theta}{\partial x} + \frac{q}{\Omega^2} \frac{\partial \theta}{\partial \tau} &= 0 \end{aligned} \quad (12)$$

We emphasize that the system of equations (12) has an interesting and important peculiarity. Propagation of fields Ω_2 and Ω_3 occurs independent of Ω_1 and Ω_4 and is described by propagation equations for three-level-atom medium in conditions of dark-state formation [1]. The fields Ω_1 and Ω_4 are described by propagation equations for two-level-atom medium [42]. This peculiarity is important because both problems are studied in sufficient detail in the literature and have analytical solutions. In particular, we can realize all phenomena taking place in the usual lambda systems with the three-level 2-3-4 system which is supported by two-level 1-2 and 4-5 systems pumping level 2 and depleting level 4, respectively. As an example, we obtain, in the considered five-level system, propagation of the adiabaton [45] in the five-level system. Fig. 6 visualizes this phenomenon (details in figure caption).

Equations (12) are valid if state $|\lambda_1\rangle$ is formed on entire length of the medium. This requires fulfillment of two conditions: i) the detunings $|\Delta_i| = \Delta$ for all i and Rabi frequencies $\Omega_1 = \Omega_4$ and ii) the adiabaticity of interaction in all of the medium. Let us examine when these conditions are met. Equations (12) show that detunings Δ_2 and Δ_3 are preserved during propagation (as they should be in a three-level system), whereas Δ_1 and Δ_4 can vary with propagation length because of self-phase modulation (as in two-level system), but, as shown in [42], these variations can be neglected if we limit the length by

$$\frac{qx}{\Delta} \frac{1}{\Delta T} \ll 1, \quad (13)$$

On the same length we can take $\Omega_1 = \Omega_4$ (adiabatic approximation for two-level system). As follows from the results of cited works the adiabaticity of interaction in two-level system breaks at the lengths when $(qx/\Delta^2 T) \sim 1$, whereas the interaction adiabaticity in three-level medium does not break at all. Another condition imposed on the length requires non-depletion of pump pulse in three-level medium for an effective population transfer[43]

$$\frac{qx}{\Delta} \frac{\Delta}{\Omega^2 T} \sim 1 \quad (14)$$

It follows from (13) and (14) that the influence of medium is determined by the factor qx/Δ , times the adiabaticity conditions for a single atom. This means that it is sufficient to require the medium parameter qx/Δ to not exceed unity by much. If we express this parameter in terms of the linear coefficient of absorption of medium α_0 , we obtain restriction for the optical length in the form:

$$\frac{qx}{\Delta} = \alpha_0 x \frac{\Gamma}{\Delta} \sim 1 \quad (15)$$

with Γ being the maximum of the relevant widths.

So, in the case of a large one-photon detuning the length of adiabaticity of interaction can exceed the length of linear absorption in medium several times. On this

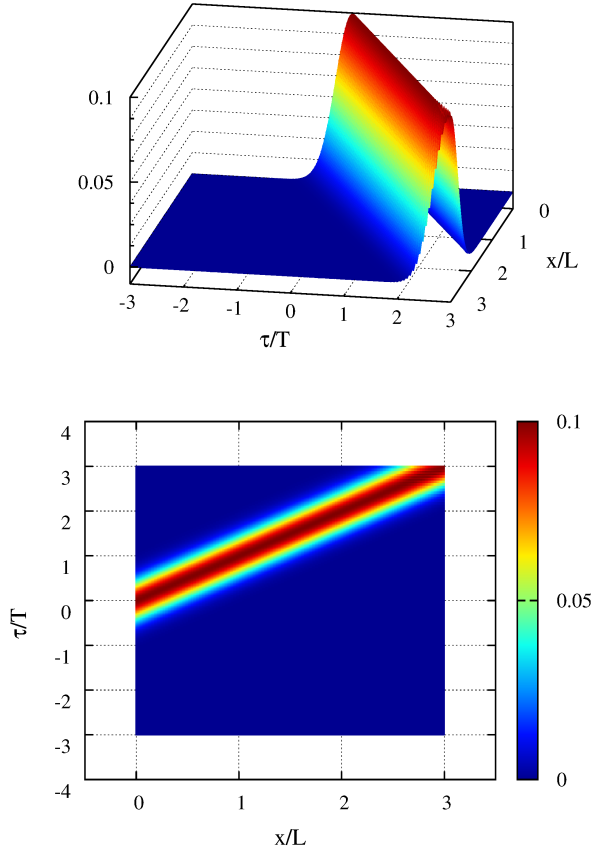


FIG. 6. Distortion-free propagation of the signal pulse Ω_2 at the subluminal group velocity. Shapes of pulses are chosen to be $\Omega_1 T = \Omega_4 T = \Omega_3 T = 30e^{(-0.2(\tau/T)^2)}$, $\Omega_2 T = 0.1e^{(-5(\tau/T)^2)}$ (linear case). The single-photon detuning is $\Delta = 100/T$. The scaled length $L = \Omega_0^2(\tau = 0)T/q$ and the time delay in medium is $\Delta t/T = x/L$

length equations (12) can be solved analytically and the

solution has the form:

$$\begin{aligned} \Omega_1 &= \Omega_4 = \Omega_{10}(\tau), \\ \Omega_2 &= \Omega_0(\tau) \sin \theta_0(\xi(x, t)), \\ \Omega_3 &= \Omega_0(\tau) \cos \theta_0(\xi(x, t)) \end{aligned} \quad (16)$$

where Ω_{10} , Ω_0 , θ_0 are the boundary conditions given at the entrance of the medium and $\xi(x, t)$ is an implicit function defined by the following expression

$$\int_{\xi}^{\tau} \Omega_0^2 dt = qx \quad (17)$$

We note, that all the above is true only if the dressed state $|\lambda_1\rangle$ is realized in the medium. In case then, as a result of an adiabatic interaction, the other dressed state (for example $|\lambda_2\rangle$) is realized, the propagation equations (11) are no longer split, and finding their solution requires an additional investigation.

V. LIGHT STORAGE

It follows from the solution (16) that, after turning off all pulses, the coherence ρ_{15} induced by these pulses remains in the medium (like in three level system):

$$\rho_{15} = -\sin \theta(\xi) \cos \theta(\xi) \quad (18)$$

where function $\xi(x)$ is defined by the following expression:

$$\int_{\xi}^{\infty} \Omega_0^2 dt = qx \quad (19)$$

Fig. 7 shows x -dependence of the ξ -function and coherence ρ_{15} after all pulses are turned off, together with the input shape of the probe pulse. The figure demonstrates that the distribution of coherence along x mirrors the arbitrary t -shape of the probe pulse at medium entrance. It is also apparent that the ξ -function has two asymptotes, $x = 0$ and $x = x_{max}$. Existence of the maximal length of medium, i.e., the length where probe pulse disappears, is the essence of light storage phenomenon. For realization of storage (and mapping of t -dependence onto x -distribution) the medium must be not shorter than x_{max} . It follows from (19) that the maximal length is representable in the form $x_{max}N = N_{ph}$, where N is the number density of resonant atoms in medium and N_{ph} is the overall photon fluxes in control and probe pulses at the medium input:

$$N_{ph} = \int_{-\infty}^{\infty} (cE_{p0}^2/\hbar\omega_p + cE_{s0}^2/\hbar\omega_s) dt \quad (20)$$

Thus, in order to write completely a light pulse into a medium, it is necessary that the number of atoms interacting with radiation be comparable with the total number of relevant photons. We emphasize that x_{max} does

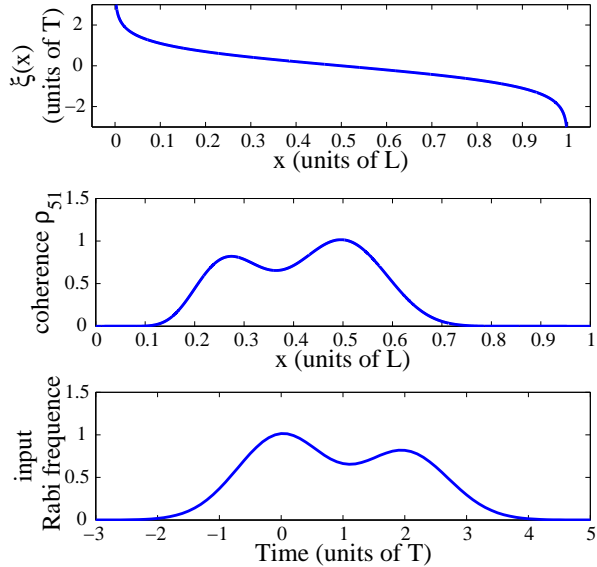


FIG. 7. The x -dependence of $\xi(x)$ -function and the spatial distribution of coherence ρ_{51} after the interaction is switched off (top) and the temporal profile of the pulse Ω_2 at the medium input (bottom).

not depend on Ω_1 and Ω_4 (the latter enters only the adiabaticity condition). Note that in linear approximation in Ω_2 we can construct a dark-state polariton similar to that in lambda system [7].

Fig. 8 shows, by means of numerical solution of corresponding equations with the use of Lax-Wendroff method [46, 47], the process of writing of a light pulse into a medium. The described process of light storage is more visualisable in the generalized lambda-system, but has no principle advantages as compared to the usual lambda system. In contrary, the M-system is much more interesting because it enables double storage, i.e., we can write two different pulses, one after another with possibility of subsequent retrieval in any desired succession. Indeed, during the whole interaction time (and also after the first writing), the coherence ρ_{31} remains zero and the population of the level 1 is close to unity (in linear approximation in Ω_2). This means that the same medium is ready for the usual lambda-storage of the second pulse. For example, we can write the pulse Ω_1 into the same medium. For this purpose the pulses Ω_1 and Ω_2 should be divided into two beams before the first storage attempt. The weak portion of Ω_1 and the strong portion of Ω_2 should be sent to a delay line, to be used for the second storage (using Ω_2 as a control pulse).

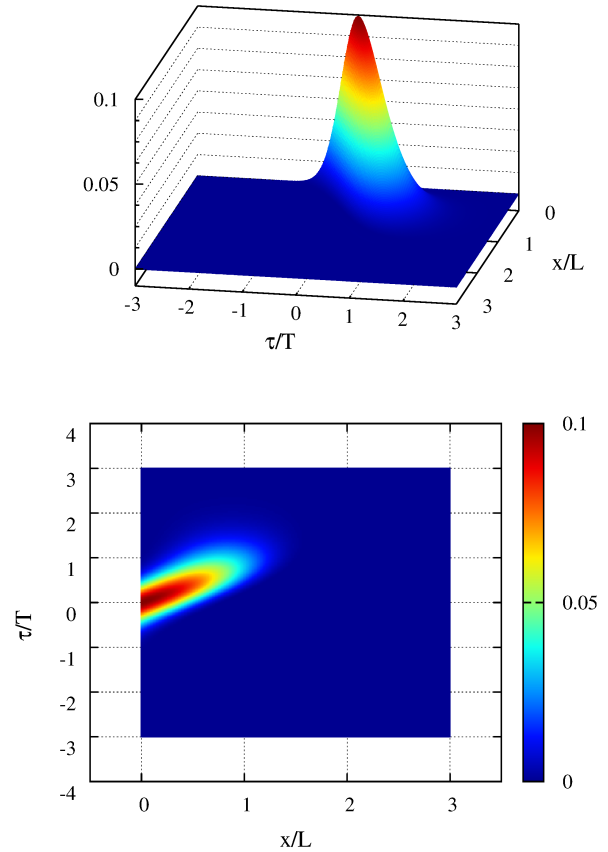


FIG. 8. Propagation of the signal pulse Ω_2 (light storage). Shapes of pulses are chosen to be $\Omega_1 T = \Omega_4 T = 30e^{-(\tau/T)^2}$, $\Omega_3 T = 30e^{-(\tau/T)^2}$, $\Omega_2 T = 0.1e^{-(\tau/T)^2}$. The single-photon detuning is $\Delta = 100/T$. The group velocity $u = c/(1 + qc/\Omega_3^2) \rightarrow 0$ when $\Omega_3^2 \rightarrow 0$

VI. CONCLUSION

We considered the behavior of a five-level atomic system and a medium of such systems driven by four laser pulses of different amplitudes and frequencies. We showed for such a system the possibility of analytical determination of system eigenfunctions and eigenvalues in case where all two-photon detunings are zero. We have obtained that, in addition to the traditional zero eigenvalue, there exists a non-zero one, for which the propagation equations in the medium are split into the equations for two- and three-level system media, i.e., two of the four laser pulses travel independently of two other. This splitting is caused by the fact that in this case the five-level system reduces to a certain effective “lambda”-system whose ground states are superpositions of two states. We derive the dressed states and dressed energies of the system, as well as conditions for adiabatic evolution, and show that the length of medium where adiabaticity is preserved exceeds several times the lin-

ear absorption length. We show that adiabatic passage permits reversible transfer of atomic population from an initial to a target state, and back again. The obtained mechanism of the population transfer may be employed for excitation of Rydberg states in atoms. We analysed the traveling of pulses in the medium and obtained, in particular, adiabaton (distortion-free) propagation at the group velocity lower than c . Also the process of information storage in five-level medium was examined. We propose a possibility of double storage of light pulses in the same medium with subsequent retrieval of the two stored pulses in desired sequence. We note that the relaxation processes have not been taken into account throughout the work. Allowance for these processes requires separate investigation. Finally we note that the considered five-level systems can experimentally be realized in a number of media, such as hyperfine structures of D-lines of alkali-metal atoms, in optical transitions of rare-earth-ion impurities in crystal matrices, in rovibrational levels of different electronic states in molecules, in problems of population transfer in entangled three two-level atoms and so on.

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APPENDIX A

Equation for eigenvalues of the interaction Hamiltonian, i.e., the equation $\det(H - \lambda I) = 0$, has, under conditions $\delta_2 = \delta_4 = 0$ and $\delta_1 = \delta_3 = \Delta$, the following form:

$$\lambda^2(\lambda - \Delta)[\lambda(\lambda - \Delta) + \Omega_s^2] + V^4\lambda = 0 \quad (21)$$

where $\Omega_s^2 = \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2$ and $V^4 = \Omega_2^2\Omega_4^2 + \Omega_1^2\Omega_3^2 + \Omega_1^2\Omega_4^2$. With the notation $x = \lambda(\lambda - \Delta)$ the equation above becomes $\lambda[x^2 - \Omega_s^2x + V^4] = 0$ and the eigenvalues are obtained directly:

$$\begin{aligned} \lambda_0 &= 0, \\ \lambda_{3,1} &= \frac{1}{2}[\Delta \pm (\Delta^2 + 4x_1)^{1/2}], \\ \lambda_{4,2} &= \frac{1}{2}[\Delta \pm (\Delta^2 + 4x_2)^{1/2}] \end{aligned} \quad (22)$$

where $x_{2,1} = (1/2)[\Omega_1^2 \pm (\Omega_s^4 - 4V^4)^{1/2}]$. We note that the condition $\Omega_s^4 \geq 4V^4$ is always met.

Conditions of interaction adiabaticity for a single atom, $|\lambda_i - \lambda_j|T \gg 1$ for any $i \neq j$ with T being the time of interaction, leads to following requirement imposed on the parameters of pulses.

$$\begin{aligned} \frac{(x_2 - x_1)T}{(\Delta^2 + 4x_2^2)^{1/2}} &\gg 1, \\ (\Delta^2 + 4x_1)^{1/2}T &\gg 1, \\ \frac{x_{1,2}T}{(\Delta^2 + 4x_{1,2})^{1/2}} &\gg 1 \end{aligned} \quad (23)$$

Note that the last condition can be fulfilled only for $V^4 \neq 0$, i.e., in the range of overlapping of pulses.

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